The University of Melbourne
Semester 2 Assessment, 2007

Department of Mathematics and Statistics
620-441 Integrable models
Final Examination

Exam duration: 2 hours
Reading time: 15 minutes
This paper has 2 pages, including the cover page

Instructions to invigilators:
No calculators, or external material are allowed.
One 7 page script book is will be given to each student initially.

Instructions to students:
There are 8 questions in this examination paper. Please attempt all questions.
Calculators are not allowed, and their use is not necessary anyway for a successful completion of this examination paper.

1. Consider the (stationary) Schrodinger equation. Propose a linear partial differential equation that together with the Schrodinger equation form a Lax pair. Show that the consistency of these two equations leads to KdV.

2. What is a pseudodifferential operator? Compute the square root of the Schrodinger operator of Question **1** as a pseudodifferential operator.

3. Starting directly from a pseudodifferential operator, derive the KP equation. Write the pseudodifferential operator in terms of a set of functions f_i. Write the form of the formal solution in terms of a set of functions w_i, and use a set of time variables x_i. State the condition under which KP reduces to KdV.
Note: It is sufficient to leave the KP equation in Lax form. That is, it is sufficient to leave the derived expression as a compatibility condition.

4. What is a Hirota derivative? What is a Hirota form of an integrable nonlinear partial differential equation? Write the KdV equation in Hirota form.

5. What is a τ function? Define a vertex operator such that the τ function for solitons can be expressed in terms of the action of vertex operators on a vacuum state.

6. What is the boson-fermion correspondence? Obtain the bosonic form of the state $\psi_{-5/2}|0>$. Obtain the fermionic form of the state $x_3 - \frac{x_1^3}{3}$.

7. What is the general form of a solution of the KP equation? Calculate the polynomial solution of the KP equation that corresponds to the generator

$$X = a\psi_{-1/2}\psi^{\star}_{-3/2} + b\psi_{-3/2}\psi^{\star}_{-1/2}$$

where a and b are constants.

8. What is an orbit of a fermionic state under $gl(\infty)$? Show that an element $|u>$ of the fermionic Fock space having charge 0 belongs to the orbit of the vacuum state if and only if

$$\sum_{i \in Z+1/2} \psi_i^{\star}|u> \otimes \psi_{-i}|u>$$