Problem Sheet 1.
Streamlines, particle paths and streamfunctions

Question 1
At time $t$, the velocity field $\mathbf{u} = u_i + v_j$ is given by
\[ u(x, y, t) = \frac{\alpha x}{1 + \alpha t}; v(x, y, t) = \beta \]
where $\alpha$ and $\beta$ are constants.

i. Find the streamlines

ii. Find the path of the particle that was located at $(X,Y)$ at $t=0$

iii. Verify from the particle path that
\[ \frac{\partial \mathbf{u}}{\partial t} \mid _{R} = \frac{D\mathbf{u}}{Dt} \]

Question 2
For the steady 2-dimensional flow
\[ u(x, y) = ay; v(x, y) = -ax \]

i. Find the streamlines

ii. Verify that
\[ \frac{\partial \mathbf{u}}{\partial t} \mid _{R} = \frac{D\mathbf{u}}{Dt} \]
and explain the answer.

iii. Find $\nabla \times \mathbf{u}$

iv. Describe the flow

Question 3
Sketch and discuss the streamline pattern for axisymmetric extensional flow,
\[ \mathbf{u} = -kx\mathbf{i} - ky\mathbf{j} + 2kz\mathbf{k} \]
and find the components of $\mathbf{u}$ in cylindrical polar coordinates.
Question 4
Derive the streamfunction and hence sketch the streamlines of a flow with velocity components

\[ u = k(x + y)i + k(x - y)j. \]

Give a possible physical interpretation of this flow.

Question 5
Show that any 2D flow of the form

\[ u_r = 0, u_{\theta} = f(r) \]

is a possible incompressible flow and find the streamlines in plane polar coordinates for the special cases

i. \( f(r) = Cr \)

ii. \( f(r) = \frac{D}{r} \)

In each case, calculate \( \nabla \times u \). Describe the flows.

Question 6
For the 2D flow represented by the streamfunction in plane polar coordinates

\[ \psi = Ur \sin \theta - Ua^2 \sin \theta / r \]

sketch the streamlines. In particular, describe the flow at large \( r \), find any stagnation points in the flow (points where \( u = 0 \)), describe the streamline \( \psi = 0 \), calculate \( \nabla \times u \) and describe what sort of flow it could possibly represent.