Problem Sheet 1. Some answers

Question 1
The particle paths are lines but the streamlines are not.

Question 2
- streamlines are circles
- \( \nabla \times \mathbf{u} = -2ak \)
- rigid body rotation about the z-axis

Question 3
Flow is axisymmetric about the z-axis but extensional along the z-axis — called uniaxial extensional flow. Flow pushes material towards the z-axis but away from the xy plane.

Question 4
\[
\psi = k\left(\frac{1}{2}y^2 + xy - \frac{1}{2}x^2\right)
\]
streamlines are hyperbolae so flow could be 2D flow into a corner.

Question 5
i. \( \psi = -\frac{1}{2}r^2 + c; \ \nabla \times \mathbf{u} = 2C; \ \text{rigid body rotation} \)
ii. \( \psi = -\log_e r + c; \ \nabla \times \mathbf{u} = 0; \ \text{irrotational flow (a ‘line vortex’)} \)

Question 6
- Flow tends to uniform flow at large \( r \)
- streamline \( \psi = 0 \) is the circle \( r = a \)
- stagnation points at \( r = a, \theta = 0, \pi \)
- \( \nabla \times \mathbf{u} = 0 \)
- potential flow about an infinitely long cylinder