Lab sheet 6. PDEs I — Method of Lines

Read the parts of Heath Chapter 11 pp. 1–16. For more detail, see ??.

We focus first on evolution PDEs with 1 spatial variable — typically parabolic or hyperbolic PDEs, although evolution PDEs with $\geq 2$ spatial variables could be treated in a similar way.

The Method of Lines is a *semi-discretization* method — the equations are discretized with respect to the spatial variable(s), leaving a system of ODEs to be solved by a suitable IVP solver. Since we know the capabilities of the Matlab IVP solvers it is natural to look at this method for solving PDEs first.

The discretization method can be any relaxation method used for BVPs e.g. finite differences, finite elements, spectral etc.

In Matlab 6, a new solver *pdepe* was introduced, that essentially uses the method of lines to solve parabolic/elliptic systems in 1 space dimension.

1 Odesuite examples

In the folder 432/odesuite, you can find 3 M-files that define functions arising from the Method of Lines. Read pp. 18–19 of the Odesuite article for some background.

*brussex.m* arises from a finite difference discretization of a coupled system of reaction-diffusion equations. Can you write down what the original equations were? What does $\alpha$ represent?

Write a driver M-file that defines $N$ and uses *ode15s* to solve this problem (see *brussex.m* for tips). To plot the result, you might use

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surf(y) or mesh(y). To change the viewing point, see help view.
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*fem1ex.m* and *fem2ex.m* arise from a Finite Element discretization of related PDEs — one appears to be the heat equation and the other the heat equation with time-dependent conductivity. This particular Galerkin discretization seems to be the same as 2nd order central differencing.

Write a driver M-file that defines $N$ and uses *ode15s* or *ode23s* to solve this problem (see *fem1ex.m*, *fem2ex.m* for tips). To plot the result, you might use

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surf(y) or mesh(y). To change the viewing point, see help view.
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2 Spectral

In the folder 432/pde/mol, you can find an M-file p27.m that uses a kind of Method of Lines to solve the Korteweg-de Vries equation

\[ u_t + uu_x + u_{xxx} = 0 \]

with periodic boundary conditions and two solitons as initial conditions.

The spatial discretization is done using Fourier basis functions, effectively by taking the Fourier transform of the PDE and using the FFT to find the numerical Fourier transform. In this case, time-stepping is done by a 4th order Runge-Kutta scheme.

3 Burger’s equation

In the folder 432/pde/mol, you can find an M-file burgmol.m that uses the Method of Lines to solve Burger’s equation

\[ u_t = cuu_x + au_{xx} \]

with Dirichlet boundary conditions.

Try "burgmol(1,1.2,.2,[ones(1,10) zeros(1,9)],0)",
"burgmol(1,0.2,.2,[ones(1,10) zeros(1,9)],0)",
"burgmol(1,0.01,.2,[ones(1,10) zeros(1,9)],0)",
"burgmol(1,0.02,.2,[ones(1,10) zeros(1,9)],0)",
"burgmol(1,0.8,.2,[ones(1,40) zeros(1,39)],0)"
to see "rarefaction wave".

Try "burgmol(1,3,80,.2,[ones(1,40) zeros(1,39)],0)"
"burgmol(1,3,80,.2,[ones(1,40) zeros(1,39)],1)"
to compare a nonstiff method to a stiff method for a relatively diffusive problem.