Qns 1–4 refer to the following information:
A random sample of twenty observations on the discrete random variable \( X \) gave the following observations:

\[
1, 0, 4, 3, 7, 2, 0, 3, 1, 2, 1, 2, 2, 5, 2, 1, 0, 1, 2, 4.
\]

1. \( x_{(14)} \) is equal to:
   A. 2
   B. 3
   C. 4
   D. 5
   E. 7

2. \( \bar{c}_{0.4} \) is equal to:
   A. 0
   B. 1
   C. 1.25
   D. 1.5
   E. 2

3. \( \hat{F}(2) \) is equal to:
   A. 0.3
   B. 0.4
   C. 0.5
   D. 0.6
   E. 0.7

4. \( \bar{x} \) is equal to:
   A. 1.85
   B. 2.15
   C. 2.45
   D. 3.15
   E. 4.30

Qns 5–7 refer to the following information:
A random sample of twenty observations on the continuous random variable \( X \) gave the following frequency table:

<table>
<thead>
<tr>
<th>observations</th>
<th>frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0–1.4</td>
<td>2</td>
</tr>
<tr>
<td>1.5–1.9</td>
<td>6</td>
</tr>
<tr>
<td>2.0–2.4</td>
<td>5</td>
</tr>
<tr>
<td>2.5–2.9</td>
<td>4</td>
</tr>
<tr>
<td>3.0–3.4</td>
<td>3</td>
</tr>
</tbody>
</table>

5. Which one of the following is correct?
   A. \( \hat{F}(1.5) = 0.4 \)
   B. \( \hat{F}(1.7) = 0.4 \)
   C. \( \hat{F}(1.9) = 0.4 \)
   D. \( \hat{F}(1.95) = 0.4 \)
   E. \( \hat{F}(2.0) = 0.4 \)

6. The sample median is approximately equal to:
   A. 1.95
   B. 2.00
   C. 2.05
   D. 2.15
   E. 2.20

7. \( \hat{f}(1.5) \) is equal to:
   A. 0.15
   B. 0.3
   C. 0.6
   D. 0.75
   E. 6

8. Which one of the following is true?
   A. \( \sum_{i=1}^{n} (\alpha x_i + \beta) = \alpha \sum_{i=1}^{n} x_i + \beta \)
   B. \( (\sum_{i=1}^{n} x_i)^2 = \sum_{i=1}^{n} x_i^2 \)
   C. \( \sum_{i=1}^{n} (x_i - \overline{x})x_i = \sum_{i=1}^{n} (x_i - \overline{x})^2 \)
   D. \( \sum_{i=1}^{n} (x_i - n\overline{x}) = 0 \)
   E. \( \sum_{i=1}^{n} (x_i - c)^2 = \sum_{i=1}^{n} x_i^2 - nc^2 \)

Qns 9–10 refer to the following information:
The table below is obtained from a sample of ten observations:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>freq(( x ))</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

9. The sample mean \( \overline{x} \) is equal to:
   A. 0.05
   B. 0.07
   C. 0.10
   D. 0.12
   E. 0.70

10. The sample variance \( s^2 \) is equal to:
    A. 0.0061
    B. 0.0068
    C. 0.0122
    D. 0.0490
    E. 0.0610
11. If $p(0) = 0.25$, the probability that, for a sample of ten observations, $\hat{p}(0) \leq 0.5$ is equal to:
   A. 0.5319
   B. 0.6231
   C. 0.9219
   D. 0.9803
   E. 0.9965

12. A random sample of $n$ observations is obtained on the random variable $X$ having cdf $F$. $\Pr(X(n) \leq x)$ is equal to:
   A. $F(x)$
   B. $F(x)^n$
   C. $[1 - F(x)]^n$
   D. $1 - F(x)^n$
   E. $1 - [1 - F(x)]^n$

13. Which one of the following is true?
   A. $\bar{x}$ is a random variable
   B. $\bar{X}$ is the realisation of $\mu$
   C. $\bar{x}$ is an estimator of $\mu$
   D. $\mu$ is the expectation of $\bar{X}$
   E. $\mu$ is an estimate of $\bar{x}$

14. A random sample of $n$ observations is obtained on $X$ which has finite positive variance $\sigma^2$. If $S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$, then:
   A. $E(S) < \sigma$
   B. $E(S) = \sigma$
   C. $E(S) > \sigma$
   D. $E(S)$ may be $<, = \sigma$ depending on the distribution of $X$.

Qns 15–17 refer to the following information:
A random sample of ten observations is obtained on $X \overset{d}{=} G(0.4)$.
If $X \overset{d}{=} G(p)$, then the pf of $X$ is given by: $p_X(x) = pq^x$, ($k = 0, 1, 2, \ldots$).

15. The probability distribution of the number of zeros in the sample is
   A. Nb(10, 0.4)
   B. Bi(10, 0.4)
   C. Bi(10, 0.6)
   D. N(4, 2.4)
   E. Pn(0.4)

16. The probability distribution of the sum of the observations in the sample is:
   A. G(0.4)
   B. Pn(4.0)
   C. Nb(10, 0.4)
   D. Bi(10, 0.4)
   E. N(15, 37.5)

17. $\Pr(X(1) \geq x)$ is equal to
   A. $0.6^x$
   B. $0.6^{x+10}$
   C. $0.6^{10x}$
   D. $0.06^x$
   E. $0.06^{x+10}$

18. A random sample of $n$ observations is obtained on $X \overset{d}{=} N(\mu, \sigma^2)$. If $n$ is large, then the variance of the sample median is approximately equal to
   A. $\frac{\pi \sigma^2}{n}$
   B. $\frac{2\sigma^2}{n}$
   C. $\frac{2\sigma^2}{\pi n}$
   D. $\frac{\pi \sigma^2}{2n}$
   E. $\frac{\sigma^2}{n}$

19. A random sample of $n$ observations is obtained on an exponential distribution with mean 1. If $n$ is large then the variance of the lower sample quartile is approximately equal to:
   A. $\frac{1}{n}$
   B. $\frac{1}{4n}$
   C. $\frac{1}{2n}$
   D. $\frac{1}{4n}$
   E. $\frac{1}{3n}$

Qns 20 and 21 refer to the following information:
A random sample of ten observations is obtained on $X \overset{d}{=} \exp(0.4)$.

20. The probability distribution of $\bar{X}$ is given by:
   A. $\gamma(10, 0.4)$
   B. exp(4)
   C. $\gamma(10, 4)$
   D. exp(40)
   E. $\gamma(10, 40)$
21. The probability distribution of \(X\) is given by:

A. \(\gamma(10, 0.4)\)
B. \(\exp(4)\)
C. \(\gamma(10, 4)\)
D. \(\exp(40)\)
E. \(\gamma(10, 40)\)

Qns 22 and 23 refer to the following information: A random sample of \(n = 16\) observations is obtained on \(X \sim N(\mu = 10, \sigma^2 = 4)\).

22. A 0.95-probability interval for \(\bar{X}\) is:

A. \(6.08 < \bar{X} < 13.92\)
B. \(8.36 < \bar{X} < 11.64\)
C. \(8.04 < \bar{X} < 11.96\)
D. \(9.02 < \bar{X} < 10.98\)
E. \(9.51 < \bar{X} < 10.49\)

23. A 0.95-probability interval for \(S^2\) is:

A. \(-6.67 < S^2 < 6.67\)
B. \(1.67 < S^2 < 7.33\)
C. \(1.72 < S^2 < 7.21\)
D. \(1.94 < S^2 < 6.67\)
E. \(1.99 < S^2 < 6.58\)

Qns 24 and 25 refer to the following information: The following diagram represents a normal probability plot:

24. \(\hat{\mu}\) is equal to

A. 0.5
B. 5
C. 10
D. 12.5
E. 15

25. \(\hat{\sigma}\) is equal to

A. 5
B. 6
C. 7.5
D. 9.5
E. 10

26. Which one of the following best describes the function of normal probability plots?

A. They make any normal cumulative frequency function a straight line due to the altered scale on the vertical axis.
B. They make any normal cumulative frequency function a straight line due to the altered scale on the horizontal axis.
C. They make any normal frequency function a straight line due to the altered scale on the vertical axis.
D. They make any normal frequency function a straight line due to the altered scale on the horizontal axis.
E. They make any probabilist appear normal.

27. If the estimator \(T_n\) is such that \(E(T_n) = \theta\) and \(\text{var}(T_n) = \frac{c}{n}\), where \(c > 0\), which one of the following is true?

A. \(T_n^2\) is consistent and unbiased for \(\theta^2\)
B. \(T_n^2\) is consistent but not unbiased for \(\theta^2\)
C. \(T_n^2\) is unbiased but not consistent for \(\theta^2\)
D. \(T_n^2\) is neither consistent nor unbiased for \(\theta^2\)
E. The consistency and unbiasedness of \(T_n^2\) for \(\theta^2\) cannot be determined from the given information.

28. A random sample of \(n\) observations is obtained on \(X \sim N(\mu, \sigma^2)\). The sample variance is given by:

\[S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2.\]

Which one of the following statements is false?

A. \(S_n^2\) is consistent for \(\sigma^2\)
B. \(S_n^2\) is unbiased for \(\sigma^2\)
C. \(S_n\) is consistent for \(\sigma\)
D. \(S_n\) is unbiased for \(\sigma\)
E. \(S_n\) is asymptotically unbiased for \(\sigma\)
29. Consider a random sample of \( n \) observations on \( X \overset{d}{=} N(\mu, \sigma^2) \). Which one of the following is a consistent estimator of \( \sigma \)?

A. \( \left( \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \right)^{1/2} \)
B. \( \frac{1}{n} \sum_{i=1}^{n} \left| X_i - \bar{X} \right| \)
C. \( \left( \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \right)^{1/2} \)
D. more than one of the above
E. none of the above

30. \( T_n \) is an unbiased estimator of \( \theta \). Which one of the following statements best represents this information?

A. As \( n \) increases, the mean of \( T_n \) approaches \( \theta \).
B. For any \( n \), the value of \( T_n \) is expected to be near to \( \theta \).
C. For any \( n \), if a large number of samples of \( n \) were obtained, the average value of \( T_n \) from these samples would be near to \( \theta \).
D. As the sample size increases, it is asymptotically certain that \( T_n \) will be close to \( \theta \).
E. The realisation of \( T_n \) must be close to \( \theta \).

31. A 95% confidence interval for \( \mu \) based on \( X \) is (4.74, 5.67). Which one of the following statements best describes this information?

A. \( \mu \) lies in the interval (4.74, 5.67) with probability 0.95.
B. \( \bar{x} \) lies in the interval (4.74, 5.67) with probability 0.95.
C. (4.74, 5.67) is the realisation of a random interval which contains \( \bar{X} \) with probability 0.95.
D. (4.74, 5.67) is the realisation of a random interval which contains \( \bar{x} \) with probability 0.95.
E. (4.74, 5.67) is the realisation of a random interval which contains \( \mu \) with probability 0.95.

32. If \( \Pr(\alpha \theta < T < b\theta) = 0.95 \), then a 95% confidence interval for \( \theta \) is given by:

A. \( \alpha t < \theta < b t \)
B. \( \frac{t}{a} < \theta < \frac{t}{b} \)
C. \( \frac{t}{b} < \theta < \frac{t}{a} \)
D. \( \frac{1}{bt} < \theta < \frac{1}{at} \)
E. \( \frac{1}{at} < \theta < \frac{1}{bt} \)

33. A random sample of 100 observations on \( X \overset{d}{=} \exp(\frac{1}{\theta}) \) gave \( \bar{x} = 3.10 \). An approximate 95% confidence interval for \( \lambda \) is:

A. (2.492, 3.708)
B. (2.592, 3.856)
C. (2.662, 3.710)
D. (2.755, 3.445)
E. (2.774, 3.465)

34. A sequence of twenty independent trials results in 16 successes. A 95% confidence interval for the probability of success is

A. 0.50 < \( p \) < 0.98
B. 0.57 < \( p \) < 0.94
C. 0.62 < \( p \) < 0.98
D. 0.65 < \( p \) < 0.94
E. 0.78 < \( p \) < 0.82

35. A sequence of 100 independent trials each having probability of success \( p \), yields 10 successes. Using an estimate of the variance of \( \hat{p} \), an approximate 95% confidence interval for \( p \) is given by:

A. (0.041, 0.159)
B. (0.051, 0.149)
C. (0.081, 0.119)
D. (0.094, 0.106)
E. (0.098, 0.102)

36. A random sample of 8 observations on \( X \overset{d}{=} R(0, \theta) \) gave: 1.33, 0.87, 5.62, 3.29, 4.03, 2.28, 3.86, 4.95
Given that \( \Pr(X_{(8)} \leq x) = (\frac{x}{\theta})^8 \), it follows that a 90% confidence interval for \( \theta \) based on \( X_{(8)} \) is:

A. (4.966, 7.850)
B. (4.982, 7.198)
C. (5.638, 6.777)
D. (5.638, 8.912)
E. (5.656, 8.173)

Qus 37 and 38 refer to the following information:
A random sample of 25 observations on \( X \overset{d}{=} N(\mu, 1) \) gave \( \bar{x} = 2.40 \).

37. A 99% confidence interval for \( \mu \) is:

A. (1.841, 2.959)
B. (1.885, 2.915)
C. (1.902, 2.898)
D. (1.934, 2.865)
E. (2.008, 2.792)
38. A 99% prediction interval for $X$ is:
   A. $(-0.227, 5.027)$
   B. $(-0.124, 4.924)$
   C. $(0.028, 4.772)$
   D. $(0.401, 4.399)$
   E. $(0.480, 4.320)$

Qus 39–41 refer to the following information:
A random sample of 16 observations on $X \sim N(\mu, \sigma^2)$ gave $\bar{x} = 2.40$ and $s = 1.20$.

39. A 95% confidence interval for $\mu$ is:
   A. $(1.761, 3.039)$
   B. $(1.764, 3.036)$
   C. $(1.767, 3.033)$
   D. $(1.812, 2.988)$
   E. $(1.874, 2.926)$

40. A 95% confidence interval for $\sigma^2$ is:
   A. $(0.655, 2.874)$
   B. $(0.749, 3.127)$
   C. $(0.786, 3.449)$
   D. $(0.821, 2.713)$
   E. $(0.864, 2.975)$

41. A 95% prediction interval for $X$ is:
   A. $(-0.236, 5.036)$
   B. $(-0.210, 5.010)$
   C. $(-0.157, 4.957)$
   D. $(-0.024, 4.824)$
   E. $(0.232, 4.568)$

Qus 42–44 refer to the following information:
random sample of five observations on $X \sim \text{Pn}(\theta)$ gave the following results:
$1, 1, 3, 2, 3$

42. The MM estimate of $\theta$ is
   A. 1.0
   B. 2.0
   C. 2.7
   D. 3.0
   E. 3.7

43. The likelihood function, $L(\theta)$, for this sample is
   A. $\frac{1}{5} e^{-5\theta} \theta^{10}$
   B. $\frac{1}{18} e^{-5\theta} \theta^{10}$
   C. $\frac{1}{24} e^{5\theta} \theta^{10}$
   D. $\frac{1}{18} e^{-10\theta} \theta^{5}$
   E. $\frac{1}{12} e^{-10\theta} \theta^{5}$

44. The ML estimate of $\theta$ is
   A. 1.0
   B. 1.8
   C. 2.0
   D. 3.6
   E. 7.2

45. If $X \sim \gamma(r, \alpha)$, then the method of moments estimator of $\alpha$, when $r$ is also unknown, based on a random sample of $n$ observations on $X$, is given by
   A. $\bar{X}$
   B. $\frac{X}{\bar{X}}$
   C. $\frac{\bar{X}^2}{\bar{X}}$
   D. $\frac{\bar{X}^2}{\bar{X}^2}$
   E. $S^2$

46. The likelihood function, $L(\theta)$, is
   A. the probability of obtaining the value $\theta$ of the parameter given the observed sample;
   B. the likelihood that the true value of the parameter is $\theta$;
   C. the probability distribution of the parameter;
   D. the likelihood that the sample is obtained;
   E. the probability of obtaining the observed sample if the true value of the parameter is $\theta$.

47. If a random sample of $n$ observations is obtained on $X$ which has pdf
   $f(x \mid \theta) = (x - \theta + 1)^{-2} (x > \theta)$
   where $\theta > 0$; then the likelihood function
   $L(\theta) = \prod_{i=1}^{n} (x_i - \theta + 1)^{-2} (\theta \in W)$
   The set $W$ is
   A. $\{\theta : \theta > 0\}$
   B. $\{\theta : \theta < x_{(1)}\}$
   C. $\{\theta : \theta > x_{(1)}\}$
   D. $\{\theta : \theta < x_{(n)}\}$
   E. $\{\theta : \theta > x_{(n)}\}$
48. The random variable $X$ has pdf

$$f(x \mid \theta) = \frac{2x}{\theta^2} \quad (0 < x < \theta)$$

A random sample of five observations on $X$ gave the following:

2.4, 0.2, 4.1, 6.3, 3.0

The ML estimate of $\theta$ is

A. 4.8
B. 6.3
C. 6.4
D. 7.6
E. 8.2

49. If $I(\theta) = n\theta^2$, then the minimum variance bound for estimation of $\frac{1}{\theta}$ is

A. $\frac{n}{\theta}$
B. $\frac{\sigma^2}{n}$
C. $\frac{\theta}{n}$
D. $\frac{1}{n}$
E. $\frac{1}{n\theta^2}$

Qns 50 and 51 refer to the following information:

A random sample of $n$ observations is obtained on $X \equiv G(\theta)$, so that $X$ has pf

$$p(x \mid \theta) = \theta(1-\theta)x^n \quad (x = 0, 1, 2, \ldots)$$

50. The ML estimator of $\theta$ is

A. $\bar{X}$
B. $\frac{1}{\bar{X}}$
C. $\frac{1}{1+X}$
D. $\frac{1+\bar{X}}{n}$
E. $\frac{\bar{X}}{1+X}$

51. The MVB for $\theta$, i.e. the lower bound for the variance of an unbiased estimator of $\theta$ is

A. $\frac{1-\theta}{n\theta^2}$
B. $\frac{1}{n\theta}$
C. $\frac{\sigma^2}{n}$
D. $\frac{\theta^2(1-\theta)}{n}$
E. $\frac{\theta(1-\theta)}{n}$

Qns 52 and 53 refer to the following information:

A coin is tossed five times to decide whether or not it is biased. If five heads or five tails are obtained then we will say it is biased, otherwise we will say it is unbiased.

52. The null hypothesis is:

A. the number of heads obtained is between 1 and 4 inclusive;
B. the coin is biased;
C. the number of heads obtained is 0 or 5;
D. the coin is unbiased;
E. the number of heads is equal to the number of tails.

53. The size of the test is equal to:

A. 0.0250
B. 0.0312
C. 0.05
D. 0.0624
E. 0.1874

54. The power of a statistical test is

A. $\Pr(\text{accept } H_0 \mid H_0 \text{ true})$
B. $\Pr(\text{reject } H_0 \mid H_0 \text{ true})$
C. $\Pr(\text{reject } H_0 \mid H_1 \text{ true})$
D. $\Pr(\text{accept } H_0 \mid H_1 \text{ true})$
E. $\Pr(\text{accept } H_0 \mid H_1 \text{ true})$

55. To test $H_0: \theta = 0.4$ against $H_1: \theta = 0.8$, we use a test statistic, $T \equiv \text{Bi}(10, \theta)$, and we reject $H_0$ if $T \geq 7$. Which one of the following statements is true?

A. size = 0.0548, power = 0.8791
B. size = 0.0548, power = 0.9991
C. size = 0.0425, power = 0.8791
D. size = 0.0425, power = 0.9991
E. size = 0.0123, power = 0.9672

56. $X \equiv \text{Bi}(20, \theta)$. To test $H_0: \theta = 0.25$ against $H_1: \theta > 0.25$ with $\alpha = 0.05$, the appropriate critical region is:

A. $X \geq 12$
B. $X \geq 11$
C. $X \geq 10$
D. $X \geq 9$
E. $X \geq 8$
57. A random sample of \( n = 5 \) observations is obtained on \( X \sim N(\mu, \sigma^2) \). To test \( H_0: \sigma^2 = 1 \) against \( H_1: \sigma^2 > 1 \), the critical region for a test of size 0.05 is:
   A. \( S^2 > 2.132 \)
   B. \( S^2 > 2.214 \)
   C. \( S^2 > 2.372 \)
   D. \( S^2 > 2.566 \)
   E. \( S^2 > 2.785 \)

58. A random sample of \( n = 20 \) observations on \( X \sim \exp(\frac{1}{2}) \) has sample mean \( \bar{X} \). To test \( H_0: \theta = 4 \) against \( H_1: \theta > 4 \), the critical region for a test of size 0.05 is:
   A. \( \bar{X} > 3.141 \)
   B. \( \bar{X} > 5.471 \)
   C. \( \bar{X} > 5.576 \)
   D. \( \bar{X} > 5.753 \)
   E. \( \bar{X} > 6.282 \)

59. A random sample of \( n = 25 \) observations on \( X \sim N(\mu, \sigma^2) \) gives \( \bar{x} = -2.5 \) and \( s = 5 \). If \( H_0: \mu = 0 \) and \( H_1: \mu \neq 0 \), then:
   A. we reject \( H_0 \) at both the 0.05 and the 0.01 levels;
   B. we reject \( H_0 \) at the 0.05 level, but accept \( H_0 \) at the 0.01 level;
   C. we reject \( H_0 \) at the 0.01 level, but accept \( H_0 \) at the 0.05 level;
   D. we accept \( H_0 \) at both the 0.05 and the 0.01 levels.

60. The likelihood ratio test is:
   A. the test with maximum power and minimum size;
   B. the test with maximum power for a given size;
   C. the test with minimum size for a given power;
   D. the test based on the maximum likelihood estimator;
   E. the test which is most likely to be correct.

61. If a random sample of \( n \) observations is obtained on \( X \) having pdf:
   \[ f(x \mid \alpha) = \frac{1}{\alpha} x^2 e^{-x/\alpha}, \quad (x > 0) \]
   then the likelihood ratio test of \( H_0: \alpha = 1 \) against \( H_1: \alpha = 2 \) is to reject \( H_0 \) if:
   A. \( \sum X_i > K \)
   B. \( \sum X_i < K \)
   C. \( 2 \sum \ln X_i - 3 \sum X_i > K \)
   D. \( 2 \sum \ln X_i - 3 \sum X_i < K \)
   E. \( \sum \ln X_i - 2 \sum X_i > K \)

Qns 62 and 63 refer to the following information:
Independent random samples of nine observations on \( X_1 \sim N(\mu_1, \sigma_1^2) \) and twenty-five observations on \( X_2 \sim N(\mu_2, \sigma_2^2) \) are obtained. These observations give:
\[
\bar{x}_1 = 17.32 \quad s_1^2 = 2.25 \\
\bar{x}_2 = 16.24 \quad s_2^2 = 2.25
\]

62. A 95% confidence interval for \( \sigma_1^2 / \sigma_2^2 \) is:
   A. (0.25, 2.76)
   B. (0.32, 2.36)
   C. (0.36, 3.95)
   D. (0.40, 3.64)
   E. (0.42, 4.12)

63. In testing \( H_0: \mu_1 = \mu_2 \) against \( H_1: \mu_1 \neq \mu_2 \), the significance level, \( P \), is such that:
   A. \( P < 0.01 \)
   B. \( 0.01 < P < 0.025 \)
   C. \( 0.025 < P < 0.05 \)
   D. \( 0.05 < P < 0.10 \)
   E. \( P > 0.10 \)

64. If \( F \equiv F_{m,n} \), then \( E(F) \) is:
   A. 1
   B. \( \frac{m+n+2}{m} \)
   C. \( \frac{n}{n-2} \)
   D. \( \frac{m+n}{m} \)
   E. \( \frac{m-2}{m} \)

65. In order that
   \[
   \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \leq t_{n_1+n_2-2}
   \]
   where \( S^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \), which one of the following conditions is not necessary?
   A. the samples on \( X_1 \) and \( X_2 \) are independent;
   B. \( X_1 \) and \( X_2 \) are normally distributed;
   C. \( \sigma_1^2 = \sigma_2^2 \);
   D. \( n_1 \) and \( n_2 \) are large;
   E. none of the above.

66. If \( F \equiv F_{6,12} \), which one of the following is true?
   A. \( \Pr(0.186 < F < 3.73) = 0.95 \)
   B. \( \Pr(0.268 < F < 3.73) = 0.95 \)
   C. \( \Pr(0.250 < F < 3.00) = 0.95 \)
   D. \( \Pr(0.186 < F < 5.37) = 0.95 \)
   E. \( \Pr(0.268 < F < 5.37) = 0.95 \)
67. The results of \( k \) independent samples gave the following values:

\[
\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 = 86 \\
\sum_{i=1}^{k} \frac{x_{i.}^2}{n_i} = 53 \\
\frac{x_{..}^2}{N} = 40
\]

The between groups sum of squares, \( B \) is
A. 13
B. 20
C. 33
D. 46
E. 53

Qns 68 and 69 refer to the following information:
Consider the analysis of variance table:

<table>
<thead>
<tr>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>between groups</td>
<td>2 *</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>within groups</td>
<td>*</td>
<td>5</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>25</td>
<td></td>
</tr>
</tbody>
</table>

in which the stars denote missing values.

68. The value of the \( F \)-statistic is
A. 2
B. 5
C. 10
D. 15
E. 20

69. The value of \( s^2 \) is
A. 10
B. 5
C. 2
D. 1
E. 0.5

Qns 70 and 71 refer to the following information:
In testing the equality of the means of five normal populations, samples of 3, 5, 4, 2 and 3 are obtained on these populations, giving the following sums of squares:

<table>
<thead>
<tr>
<th></th>
<th>between groups SS ( B = 60 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>within groups SS ( W = 12 )</td>
</tr>
<tr>
<td></td>
<td>total SS ( T = 72 )</td>
</tr>
</tbody>
</table>

70. The value of the \( F \)-statistic is such that
A. \( 0 < F < 4 \)
B. \( 4 < F < 8 \)
C. \( 8 < F < 12 \)
D. \( 12 < F < 16 \)
E. \( F > 16 \)

71. The \( F \)-statistic has degrees of freedom
A. 4, 5
B. 4, 12
C. 4, 13
D. 5, 12
E. 12, 5

Qns 72 and 73 refer to the following information:
In an experiment to compare the means of six populations, five observations are obtained on each of the six populations. It can be assumed that the populations are normally distributed with common variance \( \sigma^2 \). It is found that \( s^2 = 2.25 \).

72. Using Fisher’s method of pairwise comparisons, a 95% confidence interval for the difference \( \mu_1 - \mu_2 \) is given by \( \bar{x}_{1.} - \bar{x}_{2.} \pm u \) where \( u \) is
A. 1.38
B. 1.96
C. 2.00
D. 2.93
E. 4.15

73. Using Tukey’s method of pairwise comparisons, a 95% confidence interval for the difference \( \mu_1 - \mu_2 \) is given by \( \bar{x}_{1.} - \bar{x}_{2.} \pm v \) where \( v \) is
A. 1.38
B. 1.96
C. 2.00
D. 2.93
E. 4.15

74. A \( \chi^2 \) goodness of fit test for normality is carried out. Fifteen classes, each having expected frequencies greater than ten were used. The computed value of the \( \chi^2 \) statistic is given by \( u = 24.0 \). Using a test of size 0.05:
A. \( u \) is significantly large, so the normal model is rejected;
B. \( u \) is significantly large, so the normal model is accepted;
C. \( u \) is not significantly large, so the normal model is rejected;
D. \( u \) is not significantly large, so the normal model is accepted.
75. Consider the following contingency table:

<table>
<thead>
<tr>
<th>T</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>35</td>
</tr>
<tr>
<td>S</td>
<td>15</td>
</tr>
</tbody>
</table>

For the independence model, the expected frequencies are given by:

A. 30 20
B. 35 20
C. 30 25
D. 25 25
E. 30 30

76. In carrying out a test of independence for a $6 \times 3$ contingency table, the critical value for a test of size 0.05 is equal to

A. 18.00
B. 18.31
C. 20.48
D. 28.87
E. 31.58

77. A random sample of $n$ observations is obtained on the continuous random variable $X$. If $Z = \text{freq}(X \leq m)$ where $m$ denotes the median of $X$, then the event “$X_{(r)} < m < X_{(s)}$” is equivalent to

A. “$r < Z < s$”
B. “$r \leq Z < s$”
C. “$r < Z \leq s$”
D. “$r \leq Z \leq s$”

78. For a random sample of $n$ observations on the continuous random variable $X$, to test $H_0: c_q(X) = \xi_0$, we use the test statistic $U = \text{freq}(X \leq \xi_0)$. Under $H_0$, the distribution of $U$ is

A. $\text{Bi}(n, \xi_0)$
B. $N(\xi_0, \frac{q(1-q)}{n(\xi_0)^2})$
C. $\chi^2_{n-1}$
D. $N(q, \frac{q(1-q)}{n})$
E. $\text{Bi}(n, q)$

Qns 79 and 80 refer to the following information:

The following sample is obtained on $X$:

$\begin{array}{cccccc}
108 & 73 & 90 & 45 & 65 \\
79 & 82 & 123 & 104 & 87 \\
\end{array}$

We wish to test $H_0: \mu = 100$ against $H_1: \mu < 100$.

79. The observed value of the signed rank sum statistic, $W^+$, is

A. 6
B. 10
C. 27
D. 28
E. 45

80. The 0.05-critical value for the signed rank sum statistic, $W^+$, is

A. 8
B. 11
C. 37
D. 44
E. 47

Qns 81 and 82 refer to the following information:

We wish to test $H_0: X_1 \overset{d}{=} X_2$ against $H_1: X_2 \overset{d}{=} X_1 + a$ ($a > 0$) for the data:

$\begin{array}{cccc}
X_1 & 26 & 32 & 25 \\
X_2 & 31 & 29 & 21 & 23 & 35 \\
\end{array}$

81. The observed value of the rank sum statistic, $W$, is

A. 14
B. 15
C. 16
D. 19
E. 20

82. The 0.05-critical value for the rank sum statistic, $W$, is

A. 6
B. 7
C. 20
D. 21
E. 29
83. Data from three groups are ranked with the following results:

<table>
<thead>
<tr>
<th>Group</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>5  2 3 1</td>
</tr>
<tr>
<td>Group 2</td>
<td>4  9 7 6</td>
</tr>
<tr>
<td>Group 3</td>
<td>8 11 10 12</td>
</tr>
</tbody>
</table>

These ranks are submitted to a one-way analysis of variance on MINITAB with the following results:

<table>
<thead>
<tr>
<th>FACTOR</th>
<th>2 112.50</th>
<th>56.25</th>
<th>16.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERROR</td>
<td>9 30.50</td>
<td>3.39</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>11 143.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the value of the Kruskal-Wallis statistic, $H$?

A. 8.65  
B. 9.38  
C. 16.60 
D. 56.25 
E. 112.50 

84. The regression of $Y$ on $X$ is

A. The tendency of $Y$ to be nearer to $X$ due to the relationship between $X$ and $Y$. 
B. the function specifying the mean value of $X$ for a given value of $Y$. 
C. the relationship indicating the dependence of $Y$ on $X$. 
D. the function specifying the mean value of $Y$ for a given value of $X$. 
E. a measure of the extent to which the relationship between $X$ and $Y$ tends to regress as the variables move away from their mean values. 

85. For the given information, which one of the following is correct?

A. $\bar{x} = 4$ and $s_x = 2.02$ 
B. $\bar{x} = 4$ and $s_x = 4.26$ 
C. $\bar{x} = 6$ and $s_x = 2.02$ 
D. $\bar{x} = 4$ and $s_x = 1.42$ 
E. $\bar{x}$ and $s_x$ cannot be determined 

86. The fitted straight line regression of $Y$ on $X$ obtained using the method of least squares is given by

A. $y = 6 - x$ 
B. $y = 10 - x$ 
C. $y = 2 + x$ 
D. $y = 7 - 0.25x$ 
E. $y = 5 + 0.25x$ 

87. The sample covariance is approximately equal to

A. 22.2 
B. 4.71 
C. 2.02 
D. 1.42 
E. –2.02 

88. The correlation coefficient is approximately equal to

A. –0.8 
B. –0.5 
C. 0.0 
D. 0.5 
E. 0.8 

89. If the correlation coefficient $r_{XY}$ is not significantly different from zero, then this indicates that

A. $X$ and $Y$ are independent 
B. $X$ and $Y$ are not functionally related 
C. $X$ and $Y$ are identically distributed 
D. $X$ and $Y$ are uncorrelated 
E. $X$ and $Y$ have equal means 

90. For the data represented in the scatter diagram below, which one of the following statements about the correlation coefficient is true?

A. $0.6 < r < 1.0$ 
B. $0.2 < r < 0.6$ 
C. $-0.2 < r < 0.2$ 
D. $-0.6 < r < -0.2$ 
E. $-1.0 < r < -0.6$
Qns 91–99 refer to the following information:
Observations \( y_1, y_2, \ldots, y_{10} \) are obtained:
\[
\sum_{i=1}^{10} y_i^2 = 732.
\]
A linear model (model \( P \)) is fitted to these data giving normal equations:
\[
\begin{bmatrix}
10 & 4 \\
4 & 10
\end{bmatrix}
\begin{bmatrix}
\hat{\alpha}_1 \\
\hat{\alpha}_2
\end{bmatrix} =
\begin{bmatrix}
70 \\
-14
\end{bmatrix}
\]

91. The parameter estimates are such that:
A. \( \hat{\alpha}_1 = 8, \hat{\alpha}_2 = 4 \)
B. \( \hat{\alpha}_1 = 7/3, \hat{\alpha}_2 = -4 \frac{1}{3} \)
C. \( \hat{\alpha}_1 = 9, \hat{\alpha}_2 = -5 \)
D. \( \hat{\alpha}_1 = 7/3, \hat{\alpha}_2 = 1\frac{2}{3} \)
E. \( \hat{\alpha}_1 = 10, \hat{\alpha}_2 = -4 \)

92. \( \text{var}(\hat{\alpha}_1 - \hat{\alpha}_2) \) is equal to:
A. \( \frac{1}{4} \sigma^2 \)
B. \( \frac{1}{2} \sigma^2 \)
C. \( \frac{5}{21} \sigma^2 \)
D. \( \frac{1}{2} \sigma^2 \)
E. \( 12\sigma^2 \)

93. The model sum of squares is such that:
A. \( \text{SS} = 700, \ df = 2 \)
B. \( \text{SS} = 560, \ df = 2 \)
C. \( \text{SS} = 700, \ df = 8 \)
D. \( \text{SS} = 560, \ df = 8 \)
E. \( \text{SS} = 630, \ df = 10 \)

94. The error variance estimate, \( s^2 \), is such that:
A. \( s^2 = 2 \)
B. \( s^2 = 4 \)
C. \( s^2 = 8 \)
D. \( s^2 = 16 \)
E. \( s^2 = 32 \)

95. A 95% confidence interval for \( \alpha_1 \) is given by:
A. \( \hat{\alpha}_1 \pm 1.103 \)
B. \( \hat{\alpha}_1 \pm 1.194 \)
C. \( \hat{\alpha}_1 \pm 1.125 \)
D. \( \hat{\alpha}_1 \pm 1.458 \)
E. \( \hat{\alpha}_1 \pm 1.591 \)

Qns 96–99 refer to the following information:
For the same set of observations, another linear model (model \( Q \)) is fitted, giving normal equations:
\[
\begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 15 & 10 \\
0 & 0 & 10 & 20
\end{bmatrix}
\begin{bmatrix}
\hat{\beta}_1 \\
\hat{\beta}_2 \\
\hat{\beta}_3 \\
\hat{\beta}_4
\end{bmatrix} =
\begin{bmatrix}
36 \\
40 \\
60 \\
40
\end{bmatrix}
\]

96. The model sum of squares for model \( Q \) is such that:
A. \( \text{SS} = 716, \ df = 4 \)
B. \( \text{SS} = 716, \ df = 2 \)
C. \( \text{SS} = 724, \ df = 4 \)
D. \( \text{SS} = 724, \ df = 6 \)
E. \( \text{SS} = 748, \ df = 4 \)

97. \( \text{var}(\hat{\beta}_4) \) is equal to:
A. \( \frac{3}{400} \sigma^2 \)
B. \( \frac{1}{100} \sigma^2 \)
C. \( \frac{1}{10} \sigma^2 \)
D. \( \frac{1}{3} \sigma^2 \)
E. \( \frac{3}{1600} \sigma^2 \)

98. The degrees of freedom for testing \( H_0: \) model \( P \) vs \( H_1: \) model \( Q \) are:
A. \( 2, 2 \)
B. \( 2, 4 \)
C. \( 2, 6 \)
D. \( 4, 4 \)
E. \( 4, 6 \)

99. The \( F \)-ratio for testing \( H_0: \) model \( P \) vs \( H_1: \) model \( Q \) is equal to:
A. \( 2 \)
B. \( 3 \)
C. \( 6 \)
D. \( 8 \)
E. \( 9 \)

100. For the following data:
\[
\begin{array}{cccccc}
 x & 1 & 2 & 3 & 4 & 5 \\
 y & 1 & 4 & 8 & 12 & 16 \\
 & 2 & 5 & 9 & 13 & 17 \\
 & 3 & 6 & 10 & 14 & 18 \\
 & 7 & 11 & 15 & & \\
\end{array}
\]
it is wished to test the goodness of fit of the straight line regression. Assuming independence, normality and equal variances, the null distribution of the \( F \)-statistic to test this hypothesis is
A. \( F_{1,3} \)
B. \( F_{1,13} \)
C. \( F_{3,13} \)
D. \( F_{4,12} \)
E. \( F_{4,17} \)

Answers
ADEBD DCCBB DBDAB CCDCC
BDBCE ABDCC ECBBA EBAAC
ABCCB EBBEC DDDDA DCCBB
BCDCD AAEED BBOAE BBEBB
ABADD BEBDD CDABE CACEC