1 Problem Set 1 — Orbits

1. Let \( F(x) = x^2 \). Compute the first five points of the orbit of 1/2.

2. Let \( F(x) = x^2 - 1 \). Compute \( F^2(x) \) and \( F^3(x) \).

3. Find all the real fixed points of the following functions
   (a) \( F(x) = 3x + 2 \)
   (b) \( F(x) = x^2 - 2 \)
   (c) \( F(x) = x^3 - 3x \)
   (d) \( F(x) = |x| \)
   (e) \( F(x) = x \sin x \)

4. Find the fixed points and two-cycles of the function \( F(x) = 1 - x^2 \).

The following questions correspond to the doubling map,

\[
D : [0, 1) \mapsto [0, 1)
\]

defined by

\[
D(x) = \begin{cases} 
2x & 0 \leq x < 1/2 \\
2x - 1 & 1/2 \leq x < 1 
\end{cases}
\]

which is equivalent to

\[
D(x) = 2x \mod 1.
\]

5. Discuss the orbits of the following points under \( D(x) \):
   (a) \( x_0 = 0.3 \)
   (b) \( x_0 = 0.7 \)
   (c) \( x_0 = 1/8 \)
   (d) \( x_0 = 1/7 \)
   (e) \( x_0 = 3/11 \)

6. Explain why a computer might have difficulty computing the orbit of 1/7 if you give a decimal expansion (Hint — computers store numbers in binary).

7. Write down an explicit formula for \( D^2(x) \). Draw a graph of \( D(x) \), \( D^2(x) \) and \( D^3(x) \).

8. Find all the fixed points of \( D(x) \), \( D^2(x) \) and \( D^3(x) \). How many fixed points does \( D^n(x) \) have?
The following questions correspond to the tent map, 

\[ T : [0, 1] \to [0, 1] \]

defined by

\[ T(x) = \begin{cases} 
2x & 0 \leq x \leq \frac{1}{2} \\
2 - 2x & \frac{1}{2} < x \leq 1 
\end{cases} \]

9. Sketch \( T(x) \) — the name of this function should become obvious.

10. Find a formula for \( T^2(x) \) and sketch this function.

11. Find all fixed points of \( T(x) \) and \( T^2(x) \).

12. What does the graph of \( T^n(x) \) look like and how many fixed points does it have?