Large deviations for supercritical multi-type branching processes

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Abstract

Large deviation results are obtained for the normed limit of a supercritical multi-type branching process. Starting from a single individual of type $i$, let $L[i]$ be the normed limit of the branching process, and let $Z_{k}^{\text{min}}[i]$ be the minimum possible population size at generation $k$. If $Z_{k}^{\text{min}}[i]$ is bounded in $k$ (bounded minimum growth) then we show that $P(L[i] \leq x) = P(L[i] = 0) + x^\alpha F^*[i](x) + o(x^\alpha)$ as $x \to 0$. If $Z_{k}^{\text{min}}[i]$ grows exponentially in $k$ (exponential minimum growth) then we show that $-\log P(L[i] \leq x) = x^{-\beta/(1-\beta)}G^*[i](x) + o(x^{-\beta/(1-\beta)})$ as $x \to 0$. If the maximum family size is bounded then we get $-\log P(L[i] > x) = x^{\delta/(\delta-1)}H^*[i](x) + o(x^{\delta/(\delta-1)})$ as $x \to \infty$. Here $\alpha$, $\beta$, and $\delta$ are constants obtained from combinations of the minimum, maximum and mean growth rates, and $F^*$, $G^*$ and $H^*$ are multiplicatively periodic functions.