Allocation of Channels to Cells

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Executive Summary

The performance analysis department has been asked to calculate the number of channels to allocate to a cell in the new mobile network in order to achieve a blocking probability of 0.01 under four different hypothetical traffic scenarios. This information is given below.

1. With the arrival rate 1 and the mean holding time 3, the number of channels required is 8.

2. With the arrival rate 1 and the mean holding time 20, the number of channels required is 30.

3. With the arrival rate 10 and the mean holding time 3, the number of channels required is 42.

4. With the arrival rate 10 and the mean holding time 20, the number of channels required is 221.
1 Introduction

Management has asked the performance analysis department to calculate the number of channels needed in a cell under four different hypothetical traffic scenarios devised by the forecasting department. The task is to compute the number of channels needed to achieve a blocking probability of less than 0.01 under each of the scenarios.

2 Link Model

The number of calls in a cell can be modelled using a continuous-time Markov chain with state space $\{0, 1, 2, \ldots, C\}$, where $C$ is the number of channels. The continuous-time Markov chain has transition rates

$$q(n, m) = \begin{cases} 
\lambda I(n < C) & \text{if } m = n + 1, \\
n\mu I(n > 0) & \text{if } m = n - 1, \\
-(\lambda I(n < C) + n\mu I(n > 0)) & \text{if } m = n, \\
0 & \text{otherwise},
\end{cases}$$

(1)

where $I(A)$ is the indicator function of the event $A$ (equal to 1 if the event happens and zero otherwise). Involved in this model are the assumptions that arrivals to the cell occur in a Poisson process with parameter $\lambda$ and the call holding times are exponentially distributed with parameter $1/\mu$. Both of these assumptions are standard at the call level and are accepted as reasonable by the telecommunications industry.

3 Model Analysis

The equilibrium equations for the Markov chain with transition rates (1) are

$$\pi(n)(\lambda I(n < C) + n\mu I(n > 0)) = \pi(n+1)I(n < C)(n+1)\mu + \pi(n-1)I(n > 0)\lambda,$$

(2)

which can be decomposed into

$$(\pi(n)\lambda - \pi(n+1)(n+1)\mu)I(n < C) = (\pi(n-1)\lambda - \pi(n)n\mu)I(n > 0).$$

It follows that

$$(\pi(n)\lambda - \pi(n+1)(n+1)\mu) = 0$$

and so

$$\pi(n) = \pi(0)\frac{\lambda^n}{\mu^n n!}.$$ 

Normalising, we get

$$\pi(n) = \frac{\lambda^n / (\mu^n n!)}{\sum_{i=0}^{C} \lambda^i / (\mu^i i!)}. $$

In particular, the probability that all channels in the cell is full, and hence an arriving call is blocked is

$$\pi(C) \equiv E(\lambda/\mu, C) = \frac{\lambda^C / (\mu^C C!)}{\sum_{i=0}^{C} \lambda^i / (\mu^i i!)}. $$

(3)
4 Performance Analysis

Equation (3) gives an expression for the blocking probability of a cell with $C$ channels, arrivals occurring in a Poisson process with parameter $\lambda$ and exponentially distributed mean holding times with parameter $\mu$. In fact it can be shown (see, for example Sevastyanov [2]) that the result holds even when holding times are allowed to have a general distribution.

In order to dimension the cell so that the blocking probability is less than 0.01, but without putting in more channels than necessary, for each value of $\lambda/\mu$ we wish to choose the minimum value of $C$ such that $E(\lambda/\mu, C)$ is less than 0.01. Unfortunately equation (3) is difficult to evaluate by hand. Moreover the presence of the factorials make it difficult to evaluate directly even by computer. It is well known however (see, for example Akimaru and Kawashima [1, equation (2.26a)]) that $E(\lambda/\mu, C)$ satisfies the recursion

$$E(\lambda/\mu, C) = \frac{(\lambda/\mu)E(\lambda/\mu, C - 1)}{C + (\lambda/\mu)E(\lambda/\mu, C - 1)}$$

with

$$E(\lambda/\mu, 1) = \frac{(\lambda/\mu)}{1 + (\lambda/\mu)}.$$  

The recursion (4) is easy to program. The program which I used to compute my results, taking input from the screen and writing output to the screen is attached. Using this program I calculated that

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References
