Deancorp produces sausage by blending beef head, pork chuck, mutton, and water. The cost per pound, fat per pound, and protein per pound for these ingredients are given below:

<table>
<thead>
<tr>
<th></th>
<th>Head</th>
<th>Chuck</th>
<th>Mutton</th>
<th>Moisture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat (per lb)</td>
<td>0.05</td>
<td>0.24</td>
<td>0.11</td>
<td>0</td>
</tr>
<tr>
<td>Protein (per lb)</td>
<td>0.20</td>
<td>0.26</td>
<td>0.08</td>
<td>0</td>
</tr>
<tr>
<td>Cost (in cents)</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>

Deancorp needs to produce 100 lb of sausage and has set the following goals, listed in order of priority.
Example - Winston Page 784

**Goal 1:** Sausage should consist of at least 15% protein.
**Goal 2:** Sausage should consist of at most 8% fat.
**Goal 1:** Cost per pound of sausage should not exceed 8 cents.

Formulate a Goal Programming model for the problem.
A Goal Programming Model

Let:

\(x_1\) be amount of beef head (in pounds) used,
\(x_2\) be the amount of pork chuck (in pounds) used,
\(x_3\) be the amount of mutton (in pounds) used, and
\(x_4\) be the amount of water (in pounds) used.

Goal 1: \[0.2x_1 + 0.26x_2 + 0.08x_3 \geq 15\]
Goal 2: \[0.05x_1 + 0.24x_2 + 0.11x_3 \leq 8\]
Goal 3: \[12x_1 + 9x_2 + 8x_3 \leq 80\]

s.t. \[x_1 + x_2 + x_3 + x_4 = 100\]
\[x_1, x_2 \geq 0\]
The Lexicographic linear programming problem

\[ \text{L- } \min \{ s_1^-, s_2^-, s_3^- \} \]

\[
\begin{align*}
0.2x_1 + 0.26x_2 + 0.08x_3 + s_1^- - s_1^+ &= 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 + s_2^- &= 8 \\
12x_1 + 9x_2 + 8x_3 + s_3^- &= 80 \\
x_1 + x_2 + x_3 + x_4 &= 100 \\
x_1, x_2, s_1^-, s_1^+, s_2^-, s_3^- &\geq 0 
\end{align*}
\]
The Lexicographic linear programming problem

\[ L - \min \{ s_1^- \} \]
\[ 0.2x_1 + 0.26x_2 + 0.08x_3 + s_1^- - s_1^+ = 15 \]
\[ x_1 + x_2 + x_3 + x_4 = 100 \]
\[ x_1, x_2, s_1^-, s_1^+ \geq 0 \]

**Solution:** \( x_1 = \frac{750}{13}, \ x_4 = \frac{550}{13}, \) and \( s_1^- = 0. \) There are multiple solutions, (because there are more zero reduced cost columns than number of rows), and Goal 1 is met. Since there is “tie”, we proceed to solve the Lexicographic linear programming problem using Goal 2.
The Lexicographic linear programming problem

\( \text{L- } \min \{ s^-_2 \} \)

\[
\begin{align*}
0.2x_1 + 0.26x_2 + 0.08x_3 - s^+_1 &= 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 + s^-_2 &= 8 \\
x_1 + x_2 + x_3 + x_4 &= 100 \\
x_1, x_2, s^+_1, s^-_2 &\geq 0
\end{align*}
\]

Solution: \( x_1 = 304/7, \ x_2 = 170/7, \ x_4 = 226/7, \) and \( s^-_2 = 0. \) Goal 2 is met, and there are multiple solutions, hence we move to optimise with respect to Goal 3.
Example continued

\[
\begin{align*}
L- \quad & \min \{ s_3^- \} \\
0.2x_1 + 0.26x_2 + 0.08x_3 - s_1^+ &= 15 \\
0.05x_1 + 0.24x_2 + 0.11x_3 &= 8 \\
12x_1 + 9x_2 + 8x_3 + s_3^- &= 80 \\
x_1 + x_2 + x_3 + x_4 &= 100 \\
x_1, x_2, s_1^+, s_3^- &\geq 0
\end{align*}
\]

**Solution:** Problem infeasible. Meaning, if we attempt to keep Goal 1 and Goal 2 satisfied at their optimal level, the problem becomes infeasible when Goal 3 is added in. Comment, if the constraint, \( x_1 + x_2 + x_3 + x_4 = 100 \), had been an inequality constraint, then feasible solution might be easier to obtain.