2-person Non-zero Sum Game

\[
\begin{bmatrix}
(5, 3) & (2, 6) & (1, 7) \\
(6, 2) & (3, 5) & (7, 1) \\
(4, 6) & (2, 6) & (4, 4)
\end{bmatrix}
\]

Is \( x^* = (\frac{4}{3}, \frac{1}{4}, 5\frac{1}{2}) \), \( y^* = (\frac{4}{14}, \frac{1}{4}, \frac{1}{2}) \) an equilibrium point for the game?

Show: \( x^* A y^* \geq x A y^* \) \( \forall x \in S \)

and \( x^* A y^* \geq x^* B y^* \) \( \forall y \in T \)

\[
x^* A y^* = \begin{bmatrix} \frac{4}{3}, \frac{1}{4}, 5\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 6 & 3 & 7 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \end{bmatrix}
\]

\[= \frac{175}{48}\]
\[ x^* y^* = (108x_1 + 276x_2 + 168x_3) / 48. \]

When \( x = (0,1,0) \), \( x^* y^* = \frac{276}{48} \)
and therefore \( x^* A y^* \).
Hence, \( (x^*, y^*) \) is not in equilibrium.

What about \( x^* = (0,1,0), \ y^* = (0,1,0) \)?

\[ x^* A y^* = 3 \]
\[ x^* A y^* = 2x_1 + 3x_2 + 2x_3 \]

\[ y^* B \cdot x^* B y^* = 5 \]
\[ x^* B y^* = 2y_1 + 5y_2 + y_3. \]
\( (*) \) is satisfied and therefore \( (x^*, y^*) \) is in equilibrium.
A Non-zero Sum 2-person Game Modelling Example

Firm 1 can make

- 200 big TV (Plan 1)
- 100 big TV, 100 small TV (Plan 2)

Costs: $300 - Big TV, $120 - Small TV

Firm 2 can make

- 50 big TV (Plan 1)
- 100 small TV (Plan 2)

Costs: $320 - Big TV,
$170 - Small TV.
Weekly demand:

200 Big TV (sells at $400 each)
100 Small TV (sells at $200 each)

Note: If more TV are made than there is demand, the firm sell the same proportion of what they made.

(E.g. if I makes 200 big TVs and 2 makes 50 big TVs, and demand for big TV is 200, then 1 sells \(200 \times \frac{200}{200} = 160\) and 2 sells \(50 \times \frac{200}{200} = 40\).)

TV sets not sold worth nothing, but the firms still have to pay the cost of manufacture.
### Payoff Bi-Matrix

<table>
<thead>
<tr>
<th>Plan 1</th>
<th>Plan 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Big</td>
<td>100 Small</td>
</tr>
<tr>
<td>200 Big</td>
<td>200 Big</td>
</tr>
<tr>
<td>100 Big</td>
<td>100 Big</td>
</tr>
<tr>
<td>100 Small</td>
<td>100 Small</td>
</tr>
</tbody>
</table>

### Payoff Matrix

\[
\begin{pmatrix}
(160 \times 400, & 40 \times 400) \\
(-200 \times 300, & -50 \times 300)
\end{pmatrix}
\begin{pmatrix}
(200 \times 400, & 100 \times 200) \\
(-200 \times 300, & -100 \times 170)
\end{pmatrix}
\begin{pmatrix}
(100 \times 400, & 50 \times 400) \\
(-100 \times 300, & -50 \times 320)
\end{pmatrix}
\begin{pmatrix}
(100 \times 400, & 200 \times 50) \\
(-100 \times 300, & -100 \times 170)
\end{pmatrix}
\begin{pmatrix}
(-100 \times 300, & -100 \times 120)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
(40,000, 0) \\
(20,000, 3,000)
\end{pmatrix}
\begin{pmatrix}
(8,000, 4,000) \\
(8,000, -7,000)
\end{pmatrix}
\]