Q4. The characteristic function for the game is as follows:

\[
\begin{align*}
\nu(\emptyset) &= 0 \\
\nu(A) &= 775,000 - 200,000 - 200,000 = 375,000 \\
\nu(L) &= 800,000 - 175,000 - 200,000 = 425,000 \\
\nu(P) &= 600,000 - 190,000 - 200,000 = 210,000 \\
\nu(A, L) &= 775,000 - 200,000 \\
&\quad + 800,000 - 175,000 - 200,000 = 1,000,000 \\
\nu(A, P) &= 775,000 - 200,000 \\
&\quad + 600,000 - 190,000 - 200,000 = 785,000 \\
\nu(L, P) &= 800,000 - 175,000 \\
&\quad + 600,000 - 190,000 - 200,000 = 835,000 \\
\nu(A, L, P) &= 775,000 - 200,000 \\
&\quad + 800,000 - 175,000 \\
&\quad + 600,000 - 190,000 - 200,000 = 1,410,000 \\
\end{align*}
\]

Check that the characteristic function is superadditive:

\[
\nu(A \cup B) \geq \nu(A) + \nu(B) \quad \forall A, B \text{ (disjoint subsets of } N)\]

\[
\begin{align*}
\nu(A, L) &= 1,000,000 \geq \nu(A) + \nu(L) = 375,000 + 425,000 = 800,000 \\
\nu(A, P) &= 785,000 \geq \nu(A) + \nu(P) = 375,000 + 210,000 = 585,000 \\
\nu(L, P) &= 835,000 \geq \nu(L) + \nu(P) = 425,000 + 210,000 = 635,000 \\
\nu(A, L, P) &= 1,410,000 \geq \nu(A) + \nu(L, P) = 375,000 + 835,000 = 1,210,000 \\
\nu(A, L, P) &= 1,410,000 \geq \nu(L) + \nu(A, P) = 425,000 + 785,000 = 1,210,000 \\
\nu(A, L, P) &= 1,410,000 \geq \nu(P) + \nu(A, L) = 210,000 + 1,000,000 = 1,210,000 \\
\end{align*}
\]

\[\therefore \nu \text{ is superadditive}\]
Let \((x_A, x_L, x_P)\) be an imputation in the case.

Then we have:

1. \(x_A + x_L + x_P = \sqrt{(A, L, P)} = 1,410,000\)
2. \(x_A \geq 375,000\), \(x_L \geq 425,000\), \(x_P \geq 210,000\)
3. \(x_A + x_L \geq 1,000,000\), \(x_A + x_P \geq 785,000\), \(x_L + x_P \geq 835,000\).

From 1:

\[x_A + x_L = 1,410,000 - x_P\]

Sub. in 3(i):

\[1,410,000 - x_P \geq 1,000,000\]

\[\Rightarrow\]

\[x_P \leq 410,000\]

Also from 1:

\[x_A + x_P = 1,410,000 - x_L\]

Sub. in 3(ii):

\[1,410,000 - x_L \geq 785,000\]

\[\Rightarrow\]

\[x_L \leq 625,000\]

Also from 1:

\[x_L + x_P = 1,410,000 - x_A\]

Sub. in 3(iii):

\[1,410,000 - x_A \geq 835,000\]

\[\Rightarrow\]

\[x_A \leq 575,000\]

When combined with 2, this gives us the range of values each variable can take.
But we must also consider the constraint
\[ x_A + x_L + x_F = 1,410,000 \]

Since we have 3 variables and only 1 equation, we cannot find a unique solution.

Set \( x_F = 1,410,000 - x_A - x_L \)

Since \( x_F \geq 210,000 \) (from (2))
\[ \Rightarrow x_A + x_L \leq 1,200,000. \]

So the CORE is the set:
\[ \{ (x_A, x_L, 1,410,000 - x_A - x_L) : x_A \in [375,000, 575,000], \]
\[ x_L \in [425,000, 625,000], \]
\[ x_F \in [210,000, 410,000], \]
\[ x_A + x_L \in [1,000,000, 1,200,000] \} \]

In graphical form:
Q3. 

$n = 3, \quad |A| = 0 \quad \text{or} \quad |A| = 1 \quad \text{or} \quad |A| = 2$

$|A| = 0:\quad \rho_n(A) = \frac{|A|!(n-|A|-1)!}{n!} = \frac{0!(3-0-1)!}{3!} = \frac{2!}{3!} = \frac{1}{3}$

$|A| = 1:\quad \rho_n(A) = \frac{|A|!(n-|A|-1)!}{n!} = \frac{1!(3-1-1)!}{3!} = \frac{1!}{3!} = \frac{1}{6}$

$|A| = 2:\quad \rho_n(A) = \frac{|A|!(n-|A|-1)!}{n!} = \frac{2!(3-2-1)!}{3!} = \frac{2!}{3!} = \frac{1}{3}$

$H(A) = \{3, 3L, 3P, 3L, P3\}$

$H(L) = \{3, A3, 3P3, A1P3\}$

$H(P) = \{3, A3, 3L3, A1A1\}$

$\therefore \ x_A = \sum_{A \in H(A)} \rho_n(A) [\Phi(A \cup \{A3\}) - \Phi(A)]$

\[= \frac{1}{3} (\Phi(3) - \Phi(\emptyset)) + \frac{1}{3} (\Phi(3L) - \Phi(L))
+ \frac{1}{2} (\Phi(3P) - \Phi(P)) + \frac{1}{2} (\Phi(3L3) - \Phi(L3))
\]

\[= \frac{1}{3} (375,000 - 0) + \frac{1}{3} (1,000,000 - 425,000)
+ \frac{1}{2} (785,000 - 210,000) + \frac{1}{2} (1,410,000 - 835,000)
\]

\[= \frac{375,000}{3} + \frac{575,000}{3} + \frac{575,000}{6} + \frac{575,000}{3}
\]

\[= 508,333.33\]
\[ x_L = \sum_{A \in H(L)} p_A(A) \left[ v(A \cup \{L3\}) - v(A) \right] \]
\[ = \frac{1}{3} \left( v(L) - v(\emptyset) \right) + \frac{1}{6} \left( v(A, L) - v(A) \right) + \frac{1}{6} \left( v(L, P) - v(P) \right) + \frac{1}{3} \left( v(A, L, P) - v(A, P) \right) \]
\[ = \frac{1}{3} (425,000 - 0) + \frac{1}{6} (1,000,000 - 375,000) \]
\[ + \frac{1}{6} (835,000 - 210,000) + \frac{1}{3} (1,410,000 - 785,000) \]
\[ = \frac{425,000}{3} + \frac{625,000}{6} + \frac{625,000}{6} + \frac{625,000}{3} \]
\[ = 558,333.33 \]

\[ x_P = \sum_{A \in H(P)} p_A(A) \left[ v(A \cup \{P3\}) - v(A) \right] \]
\[ = \frac{1}{3} \left( v(P) - v(\emptyset) \right) + \frac{1}{6} \left( v(A, P) - v(A) \right) + \frac{1}{6} \left( v(L, P) - v(L) \right) + \frac{1}{3} \left( v(A, L, P) - v(A, L) \right) \]
\[ = \frac{1}{3} (210,000 - 0) + \frac{1}{6} (785,000 - 375,000) \]
\[ + \frac{1}{6} (835,000 - 425,000) + \frac{1}{3} (1,410,000 - 1,000,000) \]
\[ = \frac{210,000}{3} + \frac{410,000}{6} + \frac{410,000}{6} + \frac{410,000}{3} \]
\[ = 343,333.33 \]

\[ \text{CHECK: } x_A + x_L + x_P = 508,333.33 + 558,333.33 + 343,333.33 = 1,410,000. \]

\[ \therefore \text{Shapley's solution is given by } x = (508,333\frac{1}{3}, 558,333\frac{1}{3}, 343,333\frac{1}{3}) \]

\[ \text{YES, the Shapley value does give a reasonable division of the profits } \]
\[ \text{[it lies within the core of the game.]} \]