At the beginning of each month, we need to decide whether or not to replace the current machine. So we let \( \text{TIME} \) be the stage.

In order to make the decision, we need to know the condition of the machine at the beginning of the month. Let

\[
J_T(i) \text{ be the minimum expected discounted cost at the end of month } T \text{ given that at the beginning of month } t \text{, we have a machine in condition } i, \quad i = 0, 1, 2, \ldots, 5
\]

STARTING POINT: At the beginning of month \( T \), we have no future to consider. So

\[
J_T(i) = \min \left\{ \frac{R}{1.01} c(0) \quad \text{(REPLACE MACHINE AT BEGINNING OF MONTH \( T \))} \right. \\
\left. \frac{1}{1.01} c(i) \quad \text{(DO NOT REPLACE MACHINE)} \right\}
\]

If we replace the machine at the beginning of month \( T \), we pay \( \$R \) now to replace it, and a maintenance cost of \( \$c(0) \) at the end of the month. If we decide not to replace the machine, we pay a maintenance cost of \( \$c(i) \) at the end of the month.
Then for \( t < T \) we have:

\[
J_t(i) = \min \left\{ \frac{R}{1.01} + \frac{1}{1.01} \left[ c(i) + \sum_{j \in S_0} p_{ij} J_{t+1}(j) \right], \frac{1}{1.01} \left[ c(i) + \sum_{j \in S_0} p_{ij} f_{t+1}(j) \right] \right\}
\] (REPLACE)

So if we replace the machine at the beginning of month \( t \), we pay \$R \) now to replace it, and a maintenance cost of \$c(0)\) at the end of the month. The machine will begin the next month in state \( j \) with probability \( p_{ij} \), and will have expected discounted cost \( f_{t+1}(j) \).

If we do not replace the machine at the beginning of month \( t \), we pay a maintenance cost of \$c(i)\) at the end of the month. The machine will begin the next month in state \( j \) with probability \( p_{ij} \), and will have an expected discounted cost \( f_{t+1}(j) \).

Using this recursion we can work backwards to calculate \( J_1(i_0) \), the expected discounted cost incurred during the next \( T \) months, given that we own a state \( i_0 \) machine at the beginning of the first month.

We will have

\[
J_1(i_0) = \min \left\{ \frac{R}{1.01} + \frac{1}{1.01} \left[ c(0) + \sum_{j \in S_0} p_{ij} J_2(j) \right], \frac{1}{1.01} \left[ c(i_0) + \sum_{j \in S_0} p_{ij} J_2(j) \right] \right\}
\]