The lexicographic linear programming problem is:

\[
\begin{align*}
& \text{minimize } (s_1^-, s_2^-) \\
& \text{subject to } \\
& 2x_1 + x_2 + s_1^- - s_1^+ = 14 \\
& x_1 - x_2 + s_2^- - s_2^+ = 6 \\
& x_1 + x_2 + s_1^-, s_1^+, s_2^-, s_2^+ \geq 10 \\
& x_1, x_2, s_1^-, s_1^+, s_2^-, s_2^+ \geq 0
\end{align*}
\]

Since Goal 1 is the most important, we must first solve the problem:

\[
\begin{align*}
& \text{minimize } s_1^- \\
& \text{subject to } \\
& 2x_1 + x_2 + s_1^- - s_1^+ = 14 \\
& x_1 + x_2 \leq 10 \\
& x_1, x_2, s_1^-, s_1^+ \geq 0
\end{align*}
\]

Since the first constraint is an equality constraint, we need to add an artificial variable and use the two-phase method. Constraint 2 requires the addition of a slack variable.

<table>
<thead>
<tr>
<th>(x_1)</th>
<th>(x_2)</th>
<th>(s_1^-)</th>
<th>(s_1^+)</th>
<th>(a_1)</th>
<th>(x_3)</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Restore canonical form (i.e., make the \(w\)-row of the basic variable \(a_1\) equal to zero).

\[
R_3' = R_3 + R_1
\]
Since the artificial variable $a_1$ is out of the basis (i.e., $a_1 = 0$), we may now move to Phase 2.

Since $s_2$ is non-basic, its value is 0, and we leave the minimum value for $z_1$ (i.e., $z_1 = 0$ with $s_1 = 0$).

There are multiple optimal solutions since the non-basic variables $x_5$ and $s_1$ have reduced costs of zero.

Since there is a "tie", we proceed to solve the lexicographic linear programming problem for Goal 2.
The problem for Goal 2 is:

\[ L - \min (s_2^-) \]

\[ \text{s.t.} \]

\[ \begin{align*}
2x_1 + x_2 - s_1^+ &= 14 \\
x_1 - x_2 + s_2^- + s_2^+ &= 6 \\
x_1 + x_2 &\leq 10 \\
x_1, x_2, s_1^+, s_2^-, s_2^+ &\geq 0 \\
s_1^- &= 0
\end{align*} \]

Since the first two constraints are equality constraints, we need to add an artificial variable for each and use the two-phase method. Constraint 3 requires the addition of a slack variable.

\[ \begin{array}{cccccccccc}
3) & x_1 & x_2 & s_1^+ & s_2^- & s_2^+ & a_1 & a_2 & x_3 & \text{RHS} & \text{\textcircled{2} RATIO} \\
\hline
R_2^1 = R_2 & a_1 & 2 & 1 & -1 & 0 & 0 & 1 & 0 & 14 & 7 \\
R_2^1 = R_1 - 2R_2 & a_2 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & 6 & 6 \text{ smallest} \\
R_3 = R_3 - R_3 & x_3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 10 & 10 \\
\hline
w & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 20
\end{array} \]

Restore canonical form: \[ R_4^1 = R_4 + R_1 + R_2 \]

\[ \begin{array}{cccccccccc}
R_4^1 = R_4 - 3w & a_1 & 3 & 0 & -1 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 20
\end{array} \]

\[ \begin{array}{cccccccccc}
3) & x_1 & x_2 & s_1^+ & s_2^- & s_2^+ & a_1 & a_2 & x_3 & \text{RHS} & \text{\textcircled{2} RATIO} \\
\hline
R_1^1 = \frac{1}{3} R_1 & a_1 & 0 & \boxed{3} & -1 & 2 & 2 & 1 & -2 & 2 & \frac{2}{3} \text{ smaller} \\
R_2^1 = R_2 + R_1 & a_1 & 1 & -1 & 0 & 1 & -1 & 0 & 1 & 0 & 6 \\
R_3 = R_3 - 2R_2 & x_3 & 0 & 2 & 0 & -1 & 1 & 0 & -1 & 1 & 4 \\
R_4^1 = R_4 - 3R_2 & w & 0 & 3 & -1 & -2 & 2 & 0 & -3 & 0 & 2
\end{array} \]
<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$s_1^+$</th>
<th>$s_2^-$</th>
<th>$s_3$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$y_3$</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>-1/3</td>
<td>0</td>
<td>1/2</td>
<td>3/2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>-1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>1/3</td>
<td>0</td>
<td>3/2</td>
<td>3/2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3/2</td>
<td>-1/3</td>
<td>-1/3</td>
<td>1/3</td>
<td>1</td>
<td>8/3</td>
<td>8/3</td>
</tr>
<tr>
<td>w</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the artificial variables $y_1$ and $y_2$ are out of the basis (i.e., $y_1 = y_2 = 0$) we may now move to Phase 2.

Restore the original objective function.

| $z_2$ | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 |

This tableau is already in canonical form, and there are no non-basic variables with positive reduced costs.

Thus we STOP → this is the final tableau.

Since $s_2^-$ is non-basic, its value is 0, and we have the minimum value for $z_2$ (i.e., $z_2 = 0$ with $s_2^- = 0$).

There are multiple optimal solutions since the non-basic variables $s_1^+$ and $s_3^+$ have reduced costs of zero.

However, since there are no more goals to consider, we have our final solution.

Our solution is:  
$x_1 = \frac{20}{3} = 6 \frac{2}{3}$  
$x_2 = \frac{2}{3}$

Goal 1 is satisfied.  
Goal 2 is satisfied.