(Question adapted from Winston)

Consider a machine that may be in any of the following states 0, 1, 2, . . . , 5, where state 0 represents machine in excellent condition and state 5 represents machine in poor condition. At the beginning of each month, the stage of the machine is observed, and a decision needs to be made on whether or not to replace the machine. If the machine is replaced, a new state 0 machine arrives instantaneously. It costs $R$ dollars to replace a machine. Each month that a state $i$ machine is in operation, a maintenance cost of $c(i)$ is incurred. If a machine is in state $i$ at the beginning of the month, then with probability $p_{ij}$, the machine will begin the next month in state $j$. We assume that the condition of the machine will never improve. At the beginning of the first month, we own a state $i_0$ machine. Assuming an interest rate of 1% per month, formulate a dynamic programming recursion that could be used to minimize the expected discounted cost incurred during the next $T$ months. Note that if we replace a machine at the beginning of a month, we incur a maintenance cost of $c(0)$ during the month, and with probability $p_{i0}$, we begin the next month with a state $i$ machine.