Answers to Exam, Semester 2, 1999

Please Note: Not all questions on this exam are relevant to the course for semester 2, 2001. Only answers to the relevant questions are provided.

1. (b) 35.2; 6.50.

2. (a) 10.97; (9.64, 12.29).
(b) 2.583; (1.92, 3.93).
(c) 95% CI doesn’t include 4, so reject hypothesis.

3. (a) -0.288, -3.545; $t_a$ treats the rows as independent samples, ignoring the obvious column effects. This results in much more apparent variation between the data values. A lot of this variation is explained by the column differences. $t_b$ removes this source of variation by considering only the differences within each column — this means that the standard deviation is now much smaller, and so the $t$ value much bigger.
(b) The data are clearly paired, because the two observations in each column come from the same rock sample, and each column relates to a different rock sample. So we should use $t_b$.
(c) $|t_b| > t_9(.975) = 2.262$, so reject the null hypothesis of no difference between means. We conclude that on average the S method gives a higher reading.

5. (a) Analysis of Variance for strength

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<thead>
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<th>source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>additive</td>
<td>2</td>
<td>71.49</td>
<td>35.75</td>
<td>4.40</td>
<td>0.025 &lt; P &lt; 0.05</td>
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<tr>
<td>error</td>
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<td>8.128</td>
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<td></td>
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<tr>
<td>total</td>
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<td>177.16</td>
<td></td>
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4.40 > $F_{2,13}$, so $H_0$ should be rejected.
(b) 27.00; (23.92, 30.08).
(c) (i) $c_T$ is bigger because it sets the family error rate at 0.05, causing the individual error rate to be smaller than 0.05, resulting in a wider CI than one associated with an individual error rate of 0.05.
(ii) Because any difference between means which is larger than this quantity is significant.
(iii) $c_F = 2.160$; $c_T = 2.64$; $(-8.77, -1.23)$; $(-9.61, -0.39)$.

6. (a) $y = 15.786 + 4.4643x$; $y = 24.0893 - 0.5179x + 0.5536x^2$.
(b) Choose quadratic model — explains much more of the variation, and quadratic term significant ($P < 0.001$); testing adequacy requires multiple observations for at least some $x$, so you’d need to get more observations; unlikely that a cubic or quartic would improve fit, as almost all the variation (99.7%) has already been explained.
(c) 74.3; quadratic model provides an excellent fit for $1 \leq x \leq 8$, so should be OK for $x = 10$; however, it does require extrapolation, so we can’t be sure.
7. (a) 

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<th>df</th>
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</tr>
</thead>
<tbody>
<tr>
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<td>2345.49</td>
<td>781.83</td>
<td>102.0</td>
</tr>
<tr>
<td>between thermometers</td>
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<td>59.50</td>
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<td>total</td>
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<td>2593.00</td>
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(b) 7.67; (3.63, 25.56).

(c) $7.76 > F_{3,9}(.95) = 3.86$, so there are significant differences between thermometers.

(d) (2.32, 11.18).

(e) There are significant differences between thermometers, with T1 giving a lower reading than all the others.

9. (a) $P$-value = probability of outcome as extreme as the one observed given $H_0$; a sample value which enables a decision to be made; power = probability of rejecting $H_0$ given $H_1$ true; a property of the test.

(b) $\bar{x}$ is sample mean, estimate of $\mu$, observed, a statistic; $\mu$ is population mean, usually unobservable, a parameter.

(c) Correlation = measure of association; regression of $y$ on $x$ = mean value of $y$ given $x$; regression used for prediction.

(d) CI is a statement about a parameter; PI is a statement about a future observation; PI is wider; example: CI for $\mu$, PI for $Y$.

(e) Utility — model explains a substantial proportion of the variation in $y$; $R^2$ large; adequacy — model explains nearly as much as the full model, i.e. it is a good fit; $s^2$ small.

10 (a) Randomisation is necessary for validity, especially in avoiding bias and confounding; blocking helps precision, by grouping similar experimental units into blocks.

(b) Each treatment should be applied once within each block at random. This can be done by finding a random order for the numbers 1, 2, 3 and 4, for each block.

(c) There is a significant difference between treatments ($P = 0.004$).

(d) (i) The interaction is not significant ($P = 0.650$), so factors A and B are additive, and both are significant ($P = 0.001$ and 0.023 respectively).

(ii) (3.09, 8.44) (or (2.93, 8.59) if the interactive model is used).

11. (a) $\mu(i, x) = \mu + \alpha_i + \beta x + \beta_i x$, $\mu(i, x) = \mu + \alpha_i + \beta x$ (or equivalent formulations).

(b) That the slope is the same for both methods of hanging, i.e. $\beta_1 = \beta_2$.

(c) Use the second model, since interaction is not significant: there is a significant difference between hanging methods, with TS improving meat quality by 7.26 points on average; there is a significant improvement in meat quality with age, at the rate of 0.343 points per day.

(d) $\alpha_1 = (53.81 - 61.07)/2 = 3.63$ (this value would normally be printed in the output);

$\mu(1, x) = 53.15 - 3.63 + 0.3430(\text{aged - 5}) = 49.52 + 0.3430(\text{aged - 5})$;

$\mu(2, x) = 56.78 + 0.3430(\text{aged - 5})$; 59.9.
12. (a) large $R^2$ and adjusted $R^2$, small $s$, $C_p$ close to $p$.

(b) $x_1, x_3, x_4$ looks best (followed by $x_3, x_4$); adj $R^2$ equal best, $s$ close to $s$ for model with all variables, $C_p$ close to $p$, simpler than models with more variables; utility good — explains a large proportion of the variation; adequacy good — adding extra terms unlikely to be significant.

(c) Add $x_4$; add $x_3$ ($F$ obviously significant — calculate it if necessary); add $x_1$ ($F = \frac{37 \times 2.5508^2 - 36 \times 2.3267^2}{2.3267^2} = 8.47$); adding anything else not significant, so select $x_1, x_3, x_4$. 