Answers to Exam, Semester 2, 1999

Not all questions on this exam are relevant to the course for semester 2, 2000. Only answers to the relevant questions are provided.

1. (b) 35.2; 6.50.

2. (a) 10.97; (9.64, 12.29).
(b) 2.583; (1.92, 3.93).
(c) 95% CI doesn’t include 4, so reject hypothesis.

3. (a) -0.288, -3.545; \( t_a \) treats the rows as independent samples, ignoring the obvious column effects. This results in much more apparent variation between the data values. A lot of this variation is explained by the column differences. \( t_b \) removes this source of variation by considering only the differences within each column — this means that the standard deviation is now much smaller, and so the \( t \) value much bigger.

(b) The data are clearly paired, because the two observations in each column come from the same rock sample, and each column relates to a different rock sample. So we should use \( t_b \).

(c) \( |t_b| > t_b(975) = 2.262 \), so reject the null hypothesis of no difference between means. We conclude that on average the S method gives a higher reading.

5. (a)

<table>
<thead>
<tr>
<th>Analysis of Variance for strength</th>
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</thead>
<tbody>
<tr>
<td>source</td>
</tr>
<tr>
<td>additive</td>
</tr>
<tr>
<td>error</td>
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<tr>
<td>total</td>
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4.40 > \( F_{2,13} \), so \( H_0 \) should be rejected.

(b) 27.00; (23.92, 30.08).

(c) (i) \( c_T \) is bigger because it sets the family error rate at 0.05, causing the individual error rate to be smaller than 0.05, resulting in a wider CI than one associated with an individual error rate of 0.05.

(ii) Because any difference between means which is larger than this quantity is significant.

(iii) \( c_P = 2.160; c_T = 2.64; (-8.77, -1.23); (-9.61, -0.39) \).

6. (a) \( y = 15.786 + 4.4643x; \ y = 24.0893 - 0.5179x + 0.5536x^2 \).

(b) Choose quadratic model — explains much more of the variation, and quadratic term significant \( (P < 0.001) \); testing adequacy requires multiple observations for at least some \( x \), so you’d need to get more observations; unlikely that a cubic or quartic would improve fit, as almost all the variation (99.7%) has already been explained.

(c) 74.3; quadratic model provides an excellent fit for \( 1 \leq x \leq 8 \), so should be OK for \( x = 10 \); however, it does require extrapolation, so we can’t be sure.
7. (a) | df | SS   | MS   | F    |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>between cells</td>
<td>3</td>
<td>2345.49</td>
<td>781.83</td>
</tr>
<tr>
<td>between thermometers</td>
<td>3</td>
<td>178.50</td>
<td>59.50</td>
</tr>
<tr>
<td>error</td>
<td>9</td>
<td>69.01</td>
<td>7.668</td>
</tr>
<tr>
<td>total</td>
<td>15</td>
<td>2593.00</td>
<td></td>
</tr>
</tbody>
</table>

(b) 7.67; (3.63, 25.56).
(c) $7.76 > F_{3,9}(.95) = 3.86$, so there are significant differences between thermometers.
(d) (2.32, 11.18).
(e) There are significant differences between thermometers, with T1 giving a lower reading than all the others.

9. (a) $P$-value = probability of outcome as extreme as the one observed given $H_0$; a sample value which enables a decision to be made; power = probability of rejecting $H_0$ given $H_1$ true; a property of the test.
(b) $\bar{x}$ is sample mean, estimate of $\mu$, observed, a statistic; $\mu$ is population mean, usually unobservable, a parameter.
(c) Correlation = measure of association; regression of $y$ on $x$ = mean value of $y$ given $x$; regression used for prediction.
(d) CI is a statement about a parameter; PI is a statement about a future observation; PI is wider; example: CI for $\mu$, PI for $Y$.
(e) Utility — model explains a substantial proportion of the variation in $y$; $R^2$ large; adequacy — model explains nearly as much as the full model, i.e. it is a good fit; $s^2$ small.

10 (a) Randomisation is necessary for validity, especially in avoiding bias and confounding; blocking helps precision, by grouping similar experimental units into blocks.
(b) Each treatment should be applied once within each block at random. This can be done by finding a random order for the numbers 1, 2, 3 and 4, for each block.
(c) There is a significant difference between treatments ($P = 0.004$).
(d) (i) The interaction is not significant ($P = 0.650$), so factors A and B are additive, and both are significant ($P = 0.001$ and 0.023 respectively).
(ii) (3.09, 8.44) (or (2.93, 8.59) if the interactive model is used).

11. (a) $\mu(i, x) = \mu + \alpha_i + \beta x + \beta_i x$, $\mu(i, x) = \mu + \alpha + \beta x$ (or equivalent formulations).
(b) That the slope is the same for both methods of hanging, i.e. $\beta_1 = \beta_2$.
(c) Use the second model, since interaction is not significant: there is a significant difference between hanging methods, with TS improving meat quality by 7.26 points on average; there is a significant improvement in meat quality with age, at the rate of 0.343 points per day.
(d) $\alpha_1 = (53.81 - 61.07)/2 = 3.63$ (this value would normally be printed in the output); $\mu(1, x) = 53.15 - 3.63 + 0.3430(\text{aged} - 5) = 49.52 + 0.3430(\text{aged} - 5)$; $\mu(2, x) = 56.78 + 0.3430(\text{aged} - 5)$; 59.9.
12. (a) large $R^2$ and adjusted $R^2$, small $s$, $C_p$ close to $p$.

(b) $x_1$, $x_3$, $x_4$ looks best (followed by $x_3$, $x_4$); adj $R^2$ equal best, $s$ close to $s$ for model with all variables, $C_p$ close to $p$, simpler than models with more variables; utility good — explains a large proportion of the variation; adequacy good — adding extra terms unlikely to be significant.

(c) Add $x_4$; add $x_3$ ($F$ obviously significant — calculate it if necessary); add $x_1$ ($F = \frac{37 \times 2.5508^2 - 36 \times 2.3287^2}{2.3287^2} = 8.47$); adding anything else not significant, so select $x_1$, $x_3$, $x_4$. 