1. Consider the system of equations

\[
\begin{align*}
1.48x + 0.93y - 1.30z &= 1.03 \\
2.68x + 3.04y - 1.48z &= -0.53 \\
2.51x + 1.48y + 4.53z &= 0.05
\end{align*}
\]

(a) Solve this system of equations by Gaussian elimination, working with 3 significant figures and chopping; do not interchange rows.

(b) Repeat (a), but now use partial pivoting.

(c) Substitute the answers from (a) and (b) into the original equations, and comment on the results.

2. (a) Carry out LU factorization on the matrix:

\[
\begin{pmatrix}
4 & 12 & 8 & 4 \\
1 & 7 & 18 & 9 \\
2 & 9 & 20 & 20 \\
3 & 11 & 15 & 14
\end{pmatrix}
\]

(b) (i) Find an LU factorization of

\[
A = \begin{pmatrix}
1 & 4 & 0 & 0 \\
1 & 6 & 2 & 0 \\
0 & 6 & 10 & 3 \\
0 & 0 & 12 & 13
\end{pmatrix}
\]

and hence solve the system \(Ax = b\), where \(b = \begin{pmatrix} 5 \\ 9 \\ 19 \\ 25 \end{pmatrix}\).

(ii) Compute the maximum number of multiplications/divisions and additions/subtractions needed in solving a tridiagonal system for \(x_i, i = 1, 2, \ldots, n\). (Assume no pivoting is done.)
3. Use Cholesky factorization to solve the system of equations \( Ax = b, \)

where \( A = \begin{bmatrix}
0.05 & 0.07 & 0.06 & 0.05 \\
0.07 & 0.1 & 0.08 & 0.07 \\
0.06 & 0.08 & 0.1 & 0.09 \\
0.05 & 0.07 & 0.09 & 0.1 \\
\end{bmatrix}, \)

\( b = \begin{bmatrix}
0.23 \\
0.32 \\
0.33 \\
0.31 \\
\end{bmatrix}. \)

4. A famous ill-conditioned \( n \times n \) matrix is the Hilbert matrix

\( H_{ij} = \frac{1}{i + j - 1}, \quad i, j = 1, 2, \ldots, n \)

(a) Consider the system \( Hx = b, \)

where \( H \) is the 4x4 Hilbert matrix

and \( b^T = \begin{bmatrix}
25 & 77 & 57 & 319 \\
12 & 60 & 60 & 420 \\
\end{bmatrix} \)

for which the exact solution is

\( x^T = [1,1,1,1]. \)

Using only 3 significant digits in your arithmetic, compute a solution to \( Hx = b. \) (Hint: Decide in advance whether you are going to chop or round in order to express numbers with 3 significant digits.)

(b) Matlab can compute norms via

\( \text{norm}(A,1) \)

or \( \text{norm}(A,2) \)

or \( \text{norm}(A,\text{inf}) \)

or \( \text{norm}(A,\text{fro}) \)

Use inv (or better, invhilb) and norm to calculate a condition number for the 4 x 4 Hilbert matrix.

(c) Matlab can compute condition numbers via

\( \text{cond}(A) \) which returns the 2-norm condition number

\( \text{condest}(A) \) which returns a lower bound for the condition number in 1-norm

\( \text{rcond}(A) \) which returns an estimate for the reciprocal of the condition number in 1-norm, using a different algorithm.

Compare the results obtained using these commands with the answer you obtained in (b).