ASSIGNMENT 5: INITIAL VALUE PROBLEMS
DUE: Thursday, May 30, 2002

1. Solve the following initial value problems numerically:
   
   (a) \( y' = y - x^2 + 1 \) for \( 0 \leq x \leq 2 \) with the initial condition \( y(0) = 0.5 \).
   
   (b) \( y' = -20(x - 1)y \) for \( 0 \leq x \leq 5 \) with the initial condition \( y(0) = e^{-10} \).

   Use each of the following methods with step sizes of 0.05, 0.025, and 0.0125:
   
   (i) the simple Euler method
   
   (ii) a second-order Runge-Kutta method
   
   (iii) a fourth-order Runge-Kutta method.

   In each case, use the exact solution to find the errors. Comment on your results; in particular, do the accuracies of the methods agree with those expected theoretically?

2. The Matlab commands ode23 and ode45 use a method of Runge-Kutta-Fehlberg type of orders 2 and 3 and orders 4 and 5 respectively. Apply them to:

   \( y' = 2x - 2y \), where \( y(0) = e^{-4} \), for \( 0 \leq x \leq 3 \).

   Compare the ode23 results with a second-order Runge-Kutta method in terms of accuracy. Try tolerance values of \( 10^{-3} \) and \( 10^{-4} \); is the tolerance actually attained?

3. Apply the Adams-Bashforth-Moulton fourth-order predictor-corrector method with \( h = 0.125 \) to solve numerically the initial value problem

   \( y' = \frac{x - y}{2} \), \( y(0) = 1 \), for \( 0 \leq x \leq 3 \).

   Use the fourth-order Runge-Kutta procedure to obtain the required additional starting values.
   
   Compare your approximations with the values obtained from the analytical solution of the initial value problem.